

Total number of printed pages-7

3 (Sem-4/CBCS) MAT HC 2

2022

MATHEMATICS

(Honours)

Paper : MAT-HC-4026

(Numerical Methods)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer **any seven** questions : $1 \times 7 = 7$
- (a) What do you mean by an algorithm ?
 - (b) What is the underlying theorem of bisection method ?
 - (c) Write the iterative formula of secant method for solving an equation $f(x) = 0$.
 - (d) Consider the system of equations $Ax = b$. In which method, the matrix A can be decomposed into the product of two triangular matrices ?

Contd.

- (e) Name one iterative method for solving a system of linear equations.
- (f) Write the iterative formula of Newton-Raphson method to find the square root of 15.
- (g) What do you mean by interpolating polynomial ?
- (h) Show that $\Delta = E - 1$.
- (i) What do you mean by numerical differentiation ?
- (j) Write the formula for second order central difference approximation to the first derivative.

2. Answer **any four** questions : $2 \times 4 = 8$

- (a) Examine whether the fixed point iteration method is applicable for finding the root of the equation :

$$2x = \sin x + 5.$$
- (b) Define rate of convergence and order of convergence of a sequence.
- (c) Prove that $\mu = \left(1 + \frac{\delta^2}{4}\right)^{\frac{1}{2}}$ where μ and δ are average and central difference operators.

- (d) Verify that the following equation has a root on the interval (0,1) :

$$f(x) = \ln(1+x) - \cos x = 0.$$

- (e) If $P_1(x) = a_0 + a_1x$ such that $P_1(x_0) = f_0$ and $P_1(x_1) = f_1$, then obtain an expression for $P_1(x)$ in terms of x_i 's and f_i 's ($i = 0, 1$).
- (f) Show that $\delta = \nabla(1 - \nabla)^{-\frac{1}{2}}$.
- (g) What do you mean by degree of precision of a quadrature rule ? If a quadrature rule $I_n(f)$ integrates $1, x, x^2$ and x^3 exactly, but fails to integrate x^4 exactly, then what will be the degree of precision of $I_n(f)$?
- (h) Mention briefly about the use of Euler's method.

3. Answer **any three** questions : $5 \times 3 = 15$

- (a) Give a brief sketch of the method of false position.
- (b) Give the geometrical interpretation of Newton-Raphson method.

(c) Construct an algorithm for the secant method.

(d) Show that an LU decomposition is unique up to scaling by a diagonal matrix.

(e) Discuss about the advantages and disadvantages of Lagrange's form of interpolating polynomial.

(f) Given $f(2)=4$, $f(2.5)=5.5$, find the linear interpolating polynomial using Lagrange's interpolation. Hence find an approximate value of $f(2.2)$.

(g) Derive the closed Newton-Cotes quadrature formula corresponding to $n=1$. Why is this formula called trapezoidal rule?

(h) Evaluate $\int_0^1 \tan^{-1} x \, dx$ using Simpson's $\frac{1}{3}$ rd rule.

4. Answer **any three** questions: $10 \times 3 = 30$

(a) Perform five iterations of the bisection method to obtain the smallest positive root of the equation:

$$f(x) = x^3 - 5x + 1 = 0.$$

(b) Apply Newton-Raphson method to determine a root of the equation:

$$f(x) = \cos x - xe^x = 0.$$

Taking the initial approximation as $x_0 = 1$, perform five iterations.

(c) Form an LU decomposition of the following matrix:

$$A = \begin{pmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 5 & 8 & -2 \end{pmatrix}$$

(d) Find the order of convergence of the iterative method $x_{n+1} = \frac{1}{2} \left(x_n + \frac{a}{x_n} \right)$ to

compute an approximation to the square root of a positive real number a . To find the real root of $x^3 - x - 1 = 0$ near $x = 1$, which of the following iteration functions give convergent sequences?

(i) $x = x^3 - 1$

(ii) $x = \frac{x+1}{x^2}$

(e) Construct the difference table for the sequence of values :

$$f(x) = (0, 0, 0, \varepsilon, 0, 0, 0).$$

where ε is an error. Also show that —

(i) the error spreads and increases in magnitude as the order of differences is increased;

(ii) the errors in each column have binomial coefficients.

(f) Let $x_0 = -3$, $x_1 = 0$, $x_2 = e$ and $x_3 = \Pi$. Determine formulas for the Lagrange's polynomials $L_{3,0}(x)$, $L_{3,1}(x)$, $L_{3,2}(x)$ and $L_{3,3}(x)$ associated with the given interpolating points.

(g) For the function $f(x) = \ln x$, approximate $f'(3)$ using —

(i) first order forward difference, and

(ii) first order backward difference approximation formulas.

[Starting with step size $h = 1$, reduce it by $\frac{1}{10}$ in each step until convergence.]

$$5+5=10$$

(h) Solve the initial value problem :

$$\frac{dx}{dt} = 1 + \frac{x}{t}, \quad 1 \leq t \leq 2.5$$

$$x(1) = 1,$$

using Euler's method with step size $h = 0.5$ and find an approximate value of $x(2.5)$.