3 (Sem–6) MAT M 2

**2020**

# MATHEMATICS

( Major )

Paper : 6·2

***( Numerical Analysis )***

*Full Marks : 60*

Time : Three hours

***The figures in the margin indicate full marks for the questions.***

1. Answer the following questions : 1×7=7

   *(a)* If $\pi=\frac{22}{7}$ is approximated as 3·14, find the relative error and relative percentage error.

   *(b)* Define 'absolute error'.

   *(c)* Find the difference $\sqrt{2\cdot01}-\sqrt{2}$, correct to three significant figures.

(d) If $m$ and $n$ are positive integers, then show that $\Delta^m \Delta^n f(x) = \Delta^{m+n} f(x)$.

(e) Evaluate $\Delta^n\left(\frac{1}{x}\right)$, with 1 as the interval of differencing.

(f) Give the relationship between the operator $\Delta$ and the differential operator $D$.

(g) Write the general quadrature formula in numerical integration.

2. Answer the following questions : 2×4=8

(a) Find the number of significant figures in $x = 0{\cdot}3941$ whose absolute error is $0{\cdot}25 \times 10^{-2}$.

(b) Given $u_0 = 3$, $u_1 = 12$, $u_2 = 81$, $u_3 = 200$, $u_4 = 100$ and $u_5 = 8$, find $\Delta^5 u_0$.

(c) What is numerical differentiation ? Explain briefly its importance.

(d) Derive trapezoidal rule from Newton-Cotes quadrature formula.



3. Answer the following questions : 5×3=15

(a) Find the relative error for evaluation of $u = x_1 x_2$ with $x_1 = 4{\cdot}51$, $x_2 = 8{\cdot}32$ having absolute errors $\Delta x_1 = 0{\cdot}01$ in $x_1$ and $\Delta x_2 = 0{\cdot}01$ in $x_2$.

(b) Using the method of separation of symbols, prove the following :

$$(u_1 - u_0) - x(u_2 - u_1) + x^2(u_3 - u_2) - \ldots\ldots$$

$$= \frac{\Delta u_0}{1+x} - x\frac{\Delta^2 u_0}{(1+x)^2} + x^2\frac{\Delta^3 u_0}{(1+x)^3} - \ldots.$$

**Or**

Find the function whose first difference is $9x^2 + 11x + 5$.

(c) A second degree polynomial passes through the points (1, –1), (2, –1), (3, 1) and (4, 5). Find the polynomial.

**Or**

Using Lagrange's interpolation formula, find the form of the function given by :

| $x$ | : | 3 | 2 | 1 | –1 |
|---|---|---|---|---|---|
| $f(x)$ | : | 3 | 12 | 15 | –21 |

4. Answer ***any one*** part :

*(a)* *(i)* Apply Stirling's formula to find a polynomial of degree 4 which takes the following tabular values :

| $x$ | : | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $y = f(x)$ | : | 1 | −1 | 1 | −1 | 1 |

*(ii)* Using Newton's divided difference formula, construct the interpolating polynomial and hence compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x=5$ using the following data :

| $x$ | : | 0 | 2 | 3 | 4 | 7 | 9 |
|---|---|---|---|---|---|---|---|
| $y$ | : | 4 | 26 | 58 | 112 | 466 | 922 |

5+5=10

*(b)* *(i)* Use Bessel's formula to find $y(0{\cdot}12)$ from the following data :

| $x$ | : | 0 | 0·05 | 0·1 | 0·15 | 0·2 | 0·25 |
|---|---|---|---|---|---|---|---|
| $y$ | : | 0 | 0·10017 | 0·20134 | 0·30452 | 0·41075 | 0·52110 |



*(ii)* Find the value of $\int_1^5 \log_{10} x\,dx$, taking 8 subintervals, by trapezoidal rule. 5+5=10

5. Answer ***any one*** part :

*(a)* *(i)* In a machine a slider moves along a fixed straight rod. Its distance $x$ *cms* along the rod is given below for various values of time $t$ seconds. Find the velocity and acceleration of the slider when $t = 0{\cdot}3$.

| $t(sec)$ | : | 0 | 0·1 | 0·2 | 0·3 | 0·4 | 0·5 | 0·6 |
|---|---|---|---|---|---|---|---|---|
| $x(cm)$ | : | 30·13 | 31·62 | 32·87 | 33·64 | 33·95 | 33·81 | 33·24 |

*(ii)* The velocity $v$ *(km/min)* of a car which starts from rest, is given at fixed intervals of time $t$ *(min)* as follows :

| $t$ | : | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $v$ | : | 10 | 18 | 25 | 29 | 32 | 20 | 11 | 5 | 2 | 0 |

Estimate approximately the distance covered in 20 minutes. 5+5=10

(b) (i) Using Lagrange's formula and the following table, find $f'(3)$ and $f'(4)$ :

| $x$ | : | 1 | 2 | 4 | 8 | 10 |
|---|---|---|---|---|---|---|
| $f(x)$ | : | 0 | 1 | 5 | 21 | 27 |

(ii) Find an approximate value of $\log_e 7$ using Simpson's rule to the integral $\int_1^7 \frac{dx}{x}$.

5+5=10

6. Answer ***any one*** part :

(a) (i) Derive the rate of convergence of the Secant method.

(ii) Compute the root of $e^x - 3x = 0$, using bisection method, lying between 1·5 and 1·6, correct to two decimal places.

5+5=10

(b) (i) Using Newton-Raphson method, find the root of $x^4 - x - 10 = 0$, which is nearer to $x=2$, correct to three decimal places.



(ii) Find an approximate root of the equation $x^3 + x - 1 = 0$ near $x=1$, by the Regula-Falsi method, correct to two decimal places.

5+5=10

---