Total number of printed pages–16

3 (Sem–3/CBCS) MAT SE

2022

MATHEMATICS

(Skill Enhancement Course)

Answer the Questions from any one Option.

OPTION – A

Paper : MAT–SE–3014

(Computer Algebra Systems and Related Software)

OPTION – B

Paper : MAT–SE–3024

(Combinatorics and Graph Theory)

Full Marks : 50

Time : Two hours

The figures in the margin indicate full marks for the questions.

OPTION – A

Paper : MAT–SE–3014

(Computer Algebra Systems and Related Software)

1. Answer ***any four*** questions : 1×4=4

(a) What is meant by the case sensitivity of a computer algebra system software ?

(b) Use *any one* computer algebra system to write the expression e^x.

(c) A computer algebra system supports a wide range of mathematics such as linear algebra, calculus, and algebraic and differential equations etc. *(True/False)*

(d) The principal difference between a calculator with computer algebra system and one without computer algebra system is the ability to deal with equations symbolically rather than numerically. *(True/False)*

(e) Write any computer algebra systems' expression to generate the list $\{a, b, c, d\}$.

(f) In *R*-programming the variables *A* and *a* are same. *(True/False)*

(g) Which command is to be used to find the factorial of a number *N* in any computer algebra systems of your choice ?

(h) How do you run a file in *any one* of the computer algebra systems of your choice ?

2. Answer ***any three*** parts : 2×3=6

(a) Use any computer algebra system's expression to calculate the sum of the first 50 natural numbers.

(b) What is meant by 'built-in function' ? Give an example of a 'built-in function' in *any one* computer algebra systems.

(c) Write the format in any computer algebra system to plot a two-dimensional graph $y = f(x),\ x \in [x_1, x_2]$.

(d) Write *any one* computer algebra system software to design the function

$$f(x) = \sin^2 x + e^x|x| + \sqrt{x} + \frac{x^2}{2\pi}$$

(e) What is the purpose of 'Show' command in Mathematica ?

(f) Write the command to clear a set of variables v_1, v_2, v_3 in *any one* computer algebra system.

3. Answer ***any two*** parts : 5×2=10

(a) Given two functions $f(x)=4x(1-x)$, $g(x)=x$, write *any one* computer algebra system's expression to plot both the graphs and display them together in the range $x=0$ to $x=1$.

(b) Write *any one* computer algebra system's expression to find—

(i) intersection of two sets $\{a, b, c\}$ and $\{d, e, f, g\}$

(ii) sum of the elements in the list $\{a, b, c, d\}$

(iii) first derivative of the function $f(x)=5x^2+3$ at $x=1$

(iv) cross product of two vectors $ai+bj+ck$ and $pi+qj+rk$

(v) prime numbers between 3 and 9

(c) Using *any one* computer algebra system's expression, plot a three-dimensional graph of the surfaces x^2+y^2, $-x^2+y^2$ in the range $-2 \le x, y \le 2$.

To plot the points of the list $\{(0, 0, 1), (1, 0, 0), (0, 1, 0)\}$, which command is to be used ?

(d) Use *any* computer algebra system's expression to define the function

$$f(x)=\begin{cases} 2x & \text{for} \quad 0 \le x < ½ \\ 2(1-x) & \text{for} \quad ½ \le x < 1 \end{cases}$$

Plot the graph with colour option 'red'.

(e) Write a brief note on the plotting of parametric curves in *any one* computer algebra system.

(f) Write *any one* computer algebra system's expression to find the roots of the system of linear equations

$$ax+by=c, \quad px+qy=r.$$

Which command is to be used to solve the linear system in matrix form $AX=b$? Give its format.

4. Answer ***any three*** parts : 10×3=30

(a) (i) Use *any one* computer algebra system's expression to read the list $\{(a, b, c), (p, q, r), (l, m, n)\}$ in the form of a matrix. Write the commands to find the trace of the matrix, inverse of the matrix and transpose of the matrix. Also write the commands to determine the eigenvalues and eigenvectors of the matrix. 5

(ii) Write *any one* computer algebra system's expression to find the solution to the initial value problem $\frac{d^2y}{dx^2} = y \quad ; \quad y(0) = 1,$

$y'(0) = 0.$

Also write the commands to plot the solution. 5

(b) Write a note on the use of *any one* computer algebra system software as a calculator.

(c) In *any one* computer algebra system, write *five* options to customize a graph in the two-dimensional plot with their meanings. Explain with an example.

(d) Mention 10 'built-in functions' with their meanings in *any one* computer algebra system software.

(e) (i) Write a short note on the statistical software *R*. 5

(ii) Write the expressions in *R*-programming to calculate the sum, mean and product of the elements in the list $\{a, b, c, d\}$. 5

(f) (i) Using *any one* computer algebra systems' expression, generate a list of first 10 odd positive integers and, then find the sum of their squares. 5

(ii) Use *any one* computer algebra system's expression to write a program to animate the graph of $\sin x$ in which x varies from 0 to 2π in steps of 0·25. 5

(g) Write a short note describing different options of solving equations in Mathematica. Explain with proper examples.

(h) Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 3 & 2 & 2 \end{pmatrix}$, $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, and

$$b = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix}$$

Use commands to determine whether $AX = b$ is solvable or not. If solvable write commands to solve the above system of equations without using the available direct commands. [Consider any computer algebra system]

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OPTION – B

Paper : MAT-SE-3024

(Combinatorics and Graph Theory)

Answer ***either*** *in English* ***or*** *in Assamese.*

1. Answer ***any four*** from the following questions : 1×4=4

তলৰ *যিকোনো চাৰিটা* প্ৰশ্নৰ উত্তৰ কৰা ঃ

(a) If there are 28 boys and 14 girls in a class then in how many ways you can choose a captain for the class ?

শ্ৰেণী এটাৰ 28 জন লৰা আৰু 14 জনী ছোৱালীৰ পৰা কিমান ধৰণেৰে শ্ৰেণীটোৰ বাবে এজন কেপ্তেইন বাচিব পাৰিবা ?

(b) In how many ways you can arrange eight different books in a row out of which four are mathematics book and four are of physics ?

আঠখন ভিন্ন কিতাপক এটা শাৰীত কিমান ভিন্ন ধৰণে সজাব পাৰি যদি তাৰে চাৰিখন গণিতৰ আৰু চাৰিখন পদাৰ্থবিদ্যাৰ কিতাপ হয় ?

(c) State the pigeonhole principle of counting.

গণনাৰ পিজনহল সূত্ৰটো লিখা।

(d) What is Ramsey's number?

ৰামচীৰ সংখ্যা বুলিলে কি বুজা ?

(e) Define a finite graph.

এটা সসীম লেখৰ সংজ্ঞা দিয়া।

(f) What is the maximum number of edges in a simple graph with n vertices?

n টা শীৰ্ষবিন্দুৰ সৰল লেখ এটাত সৰ্বোচ্চ কিমান সংখ্যক বাহু থাকিব ?

(g) Draw a disconnected graph.

বিচ্ছিন্ন লেখ এটাৰ চিত্ৰ আঁকা।

(h) Draw a complete graph with five vertices.

পাঁচটা শীৰ্ষবিন্দুৰ সম্পূৰ্ণ লেখ এটাৰ চিত্ৰ আঁকা।

2. Answer ***any three*** from the following questions : 2×3=6

তলৰ *যিকোনো তিনিটা* প্ৰশ্নৰ উত্তৰ কৰা ঃ

(a) Draw a graph with degree sequence (2, 2, 2, 0) or explain the reason if no such graph exists.

(2, 2, 2, 0) ঘাতক্ৰমৰ শীৰ্ষ বিন্দুযুক্ত লেখ এটাৰ চিত্ৰ আঁকা আৰু যদিই স্থিত নহয় তাৰ কাৰণ ব্যাখ্যা কৰা।

(b) Show that every simple finite graph has two vertices of the same degree.

দেখুওৱা যে প্ৰতিটো সৰল সসীম লেখৰ দুটা সমান ঘাতৰ শীৰ্ষবিন্দু থাকে।

(c) If five points are chosen at random in the interior of an equilateral triangle each side of which is two units long, then show that at least one pair of points has a separation of less than one unit.

দুই একক দৈর্ঘ্যৰ বাহুবিশিষ্ট এটা সমবাহু ত্ৰিভুজৰ অন্তৰ্ভাগত যাদৃচ্ছিকভাবে যদি পাঁচটা বিন্দু লোৱা হয়, দেখুওৱা যে সিহতৰ মাজৰ অন্তত এযোৰ বিন্দুৰ মাজৰ দূৰত্ব এক এককতকৈ কম হয়।

(d) Find the number of positive integer solutions of $x+y+z=21$.

$x+y+z=21$ সমীকৰণৰ ধনাত্মক অখণ্ড সংখ্যাৰ সমাধানৰ সংখ্যা নিৰ্ণয় কৰা।

(e) Examine existence of isomorphism between the complete graph K_6 and the complete bipartite graph $K_{3,3}$.

সম্পূৰ্ণ লেখ K_6 আৰু বিপার্টাইট লেখ $K_{3,3}$ ৰ মাজত এঁকেকীসমকাৰিকতা থাকিবনে পৰীক্ষা কৰা।

(f) Define a walk and give geometrical representation of a walk.

লেখৰ এটা খোজ (ৱাক)ৰ সংজ্ঞা দিয়া আৰু পথ এটিৰ জ্যামিতিক উপস্থাপন আগবঢ়োৱা।

3. Answer ***any two*** from the following questions : 5×2=10

তলৰ *যিকোনো দুটা* প্ৰশ্নৰ উত্তৰ কৰা ঃ

(a) Find the number of positive integers less than 10000 such that the sum of digits is equal to 10.

যোগফল 10 হোৱা 10000 তকৈ সৰু ধনাত্মক অখণ্ড সংখ্যাৰ সংখ্যা নিৰ্ণয় কৰা।

(b) Show that the maximum number of edges in a complete bipartite graph of n vertices is $n^2/4$.

দেখুওৱা যে nটা শীৰ্ষ বিন্দুৰ সম্পূৰ্ণ বিপার্টাইট লেখ এটাৰ সৰ্বোচ্চ বাহুৰ সংখ্যা $n^2/4$.

(c) Prove that a self-complementary graph contains $4k$ or $4k+1$ vertices, where k is an integer.

প্ৰমাণ কৰা যে এটা স্ব-পূৰক লেখত $4k$ বা $4k+1$টা শীৰ্ষবিন্দু থাকে, য'ত k এটা অখণ্ড সংখ্যা।

(d) Prove that for any graph G with six points G or its complement contains a triangle.

প্ৰমাণ কৰা যে ছয়টা শীৰ্ষবিন্দুৰ এটা লেখ বা তাৰ পূৰক লেখত এটা ত্ৰিভুজ থাকিব।

(e) Prove that every closed odd walk in a graph contains an odd cycle.

প্ৰমাণ কৰা যে এটা লেখৰ প্ৰতিটো বন্ধ অযুগ্ম ৱাকত এটা অযুগ্ম বৰ্তনী থাকিব।

(f) Write a short note on application of graph theory in social networking.

'চ'চিয়েল নেটৱৰ্কিঙ'ত গ্ৰাফ থিওৰিৰ ব্যৱহাৰৰ ওপৰত এটা চমু টোকা লিখা।

4. Answer ***any three*** of the following questions : 10×3=30

তলৰ *যিকোনো তিনিটা* প্ৰশ্নৰ উত্তৰ কৰা ঃ

(a) Show that a connected planar graph with n vertices and e edges has $e-n+2$ regions.

দেখুওৱা যে nটা শীৰ্ষ বিন্দু আৰু eটা বাহুযুক্ত এটা সংযুক্ত সমতলীয় লেখৰ $e-n+2$ টা ক্ষেত্ৰ থাকে।

(b) Define a regular graph. Draw n-regular graphs for $n=1, 2, 3, 4$. 2+8=10

এটা নিয়মীয়া লেখৰ সংজ্ঞা দিয়া। $n=1, 2, 3, 4$ ৰ বাবে n-নিয়মীয়া লেখ বিলাকৰ চিত্ৰ আঁকা।

(c) Define a bipartite graph. When is a bipartite graph called complete ? Draw $K_{2,3}, K_{2,4}, K_{3,3}, K_{3,4}$.

1+1=8=10

বিপাৰ্টাইট লেখ এটাৰ সংজ্ঞা দিয়া। কেতিয়া এটা বিপাৰ্টাইট লেখক সম্পূৰ্ণ বুলি কোৱা হব ? $K_{2,3}, K_{2,4}, K_{3,3}, K_{3,4}$ লেখ বিলাকৰ চিত্ৰ আঁকা।

(d) Let $\cup$ be a finite set. For subsets $X, Y \subseteq \cup$, show that

$|X \cup Y| = |X| + |Y| - |X \cap Y|$.

Find number of positive integers less than 1000, which are divisible by 5 or 7. 5+5=10

$\cup$ এটা সসীম সংহতি আৰু X, Y তাৰ দুটা উপসংহতি। দেখুওৱা যে $|X \cup Y| = |X| + |Y| - |X \cap Y|$.

1000 তকৈ সৰু 5 বা 7 ৰে বিভাজ্য ধনাত্মক অখণ্ড সংখ্যা কিমান আছে নিৰ্ণয় কৰা।

(e) Prove that a connected graph contains a Eulerian circuit if and only if each of its vertices is of even degree.

প্ৰমাণ কৰা যে এটা সংযুক্ত লেখত এটা অয়লাৰ বৰ্তনী থাকিব যদি আৰু যদিহে তাৰ প্ৰতিটো শীৰ্ষবিন্দু যুগ্ম ঘাতৰ হয়।

(f) Define a Hamiltonian graph. Draw a Hamiltonian graph. If G is a connected planar graph, show that G has a vertex with degree ≤ 5. 2+2+6=10

হেমিল্টনীয় লেখ এটাৰ সংজ্ঞা দিয়া। হেমিল্টনীয় লেখ এটাৰ চিত্ৰ আঁকা। যদি G এটা সংযুক্ত সমতলীয় লেখ হয়, দেখুওৱা যে G ত এটা পাচতকৈ কম বা পাঁচটা ঘাতৰ শীৰ্ষবিন্দু থাকিব।

(g) Define subgraph of graph. Give examples of vertex-disjoint subgraph and edge-disjoint subgraph with proper diagrams. What are spanning subgraph and induced subgraph? 2+4+4=10

লেখ এটাৰ উপলেখৰ সংজ্ঞা দিয়া। শীৰ্ষবিন্দু-বিচ্ছিন্ন উপলেখ আৰু প্ৰান্ত-বিচ্ছিন্ন উপলেখৰ উদাহৰণ দিয়া। প্ৰসাৰিত উপলেখ আৰু আবেশিত উপলেখ কি ?

(h) Define walk, trail, path, cycle and connected graph with diagram. 2+2+2+2+2=10

লেখৰ খোজ (ৱাক), দৌৰ (ট্ৰেইল), পথ (পাঠ), বৰ্তনী (চাইকল) আৰু সংযুক্ত (কানেক্টেড) লেখৰ চিত্ৰসহ সংজ্ঞা দিয়া।