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**3 (Sem-6/CBCS) MAT HC 2**

**2024**

**MATHEMATICS**

(Honours Core)

Paper : MAT-HC-6026

**(Partial Differential Equations)**

Full Marks : 60

Time : Three hours

**The figures in the margin indicate full marks for the questions.**

1. Answer the following: 1×7=7

(i) Under which of the following conditions does arbitrary constant elimination usually produce more than one partial differential equation of order one ?

(a) The number of arbitrary constants is less than that of independent variables

*Contd.*



(b) The number of arbitrary constants equals the number of independent variables

(c) The number of arbitrary constants is more than that of independent variables

(d) Both (a) and (b)

*(Choose the correct answer)*

(ii) State True or False :

$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$  is a first order quasi-linear partial differential equation.

(iii) The order of  $p \tan y + q \tan x = \sec^2 z$  is \_\_\_\_\_.

(iv) The Charpit's method is used for

(a) general solution

(b) complete solution

(c) singular solution

(d) complete integral

*(Choose the correct answer)*

(v) Jacobi's auxiliary equations for  $p_1 x_1 + p_2 x_2 - p_3^2 = 0$  are \_\_\_\_\_.

(vi) What are the characteristic equations of  $u_x - u_y = u$  ?

(vii) The equation  $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = 0$  is

(a) parabolic for  $x \neq 0$  and  $y \neq 0$  only

(b) parabolic for  $x = 0$  and  $y = 0$  only

(c) parabolic everywhere

(d) parabolic nowhere

*(Choose the correct answer)*

2. Answer in short : 2×4=8

(i) Consider an equation of the form  $a(x, y, u)u_x + b(x, y, u)u_y = c(x, y, u)$ , where its coefficients  $a$ ,  $b$  and  $c$  are functions of  $x$ ,  $y$  and  $u$ . Is it linear? Justify your answer.



(ii) Eliminate the arbitrary function  $f$  from  $z = x^n f\left(\frac{y}{x}\right)$  to form a partial differential equation.

(iii) Mention when Jacobi's method is used. Name an advantage of Jacobi's method over Charpit's method.

(iv) Construct an example of a partial differential equation that is elliptic in one domain but hyperbolic in another.

3. Answer **any three** :  $5 \times 3 = 15$

(i) Find the partial differential equation that all surfaces of revolution satisfy with the  $z$ -axis as the axis of symmetry, along with a suitable explanation.

(ii) Find the general solution of the differential equation

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z.$$

(iii) Find the integral surface of the equation

$$(x - y)y^2 p + (y - x)x^2 q = (x^2 + y^2)z$$

through the curve  $xz = a^3, y = 0$ .

(iv) Reduce the equation

$$u_x + 2xyu_y = x$$

to canonical form, and obtain the general solution.

(v) Discuss the general solution of  $Au_{xx} + Bu_{xy} + Cu_{yy} = 0$  with constant coefficients in hyperbolic case.

4. Answer the following :  $10 \times 3 = 30$

(i) Discuss briefly the essential steps in Charpit's method for solving partial differential equations. Use this method to solve the equation  $p = (z + qy)^2$ .

**Or**

Show that the only integral surface of the equation  $2q(z - px - qy) = 1 + q^2$  which is circumscribed about the paraboloid  $2x = y^2 + z^2$  is the enveloping cylinder which touches it along its section by the plane  $y + 1 = 0$ .



- (ii) Describe in brief the key components of the 'method of separation of variables'. Use this method suitably to solve the equation  $u_x + u = u_y$ ,  $u(x, 0) = 4e^{-3x}$ .

**Or**

Use  $v = \ln u$  and  $v(x, y) = f(x) + g(y)$  to solve the equation  $x^2u_x^2 + y^2u_y^2 = u^2$ .

Also, discuss briefly the approach adopted to solve the above equation.

- (iii) Consider the wave equation

$$u_{tt} - c^2u_{xx} = 0, \quad c \text{ is constant.}$$

Establish that any general solution of this equation can be expressed as the sum of two waves, one travelling to the right with constant velocity  $c$  and the other travelling to the left with the same velocity  $c$ .

**Or**

Find the general solution of the following equations :

(a)  $yu_{xx} + 3yu_{xy} + 3u_x = 0, y \neq 0$

(b)  $4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$

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