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3 (Sem-4/CBCS) PHY HC 1

2024

PHYSICS

(Honours Core)

Paper : PHY-HC-4016

(Mathematical Physics-III)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions: $1 \times 7 = 7$

- (a) What is the smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$?
- (b) What is Argand diagram ?
- (c) State Taylor's theorem.

Contd.

(d) State convolution theorem of Fourier transform.

(e) Name any two branches of physics where tensors are applied.

(f) Find the Laplace transform of the function $f(t) = 1$.

(g) Write down the conditions for existence of Fourier transform.

2. Answer the following questions: $2 \times 4 = 8$

(a) Express the following complex number in polar form and plot in Argand diagram

$$2 + 2\sqrt{3}i$$

(b) Find Laplace transform of the function

$$F(t) = 3e^{3t} + 5t^4 - 4\cos 2t$$

(c) Check whether the complex function $f(z) = \frac{1}{z}$ is analytic or not.

(d) Prove that $\partial_{ij}\epsilon_{ijk} = 0$.

3. Answer any three questions of the following: $5 \times 3 = 15$

(a) Show that the real and imaginary parts of the function $w = \log z$ satisfy the Cauchy-Riemann equations when z is not zero. Find its derivative. $3 + 2 = 5$

(b) Define Fourier transform of a function $f(x)$. Find Fourier transform of $e^{-x^2/2}$. What is your inference? $1 + 3 + 1 = 5$

(c) Evaluate $\int_C (z - z^2) dz$, where C is upper half of the circle $|z| = 1$. What is the value of this integral if C is the lower half of the above circle? $3 + 2 = 5$

- (d) Using Laplace transform, find the solution of the initial value problem

$$y'' + 9y = 6 \cos 3t, \quad y(0) = 2, \quad y'(0) = 0$$

- (e) What are raising and lowering of indices of a tensor? Prove that the two operations of raising and lowering the indices are reciprocal to each other.

$$2+3=5$$

4. Answer **any three** of the following questions :

$$10 \times 3 = 30$$

- (a) (i) Obtain the Cauchy-Riemann conditions for the function $f(z) = u + iv$ to be an analytic function where u and v are the functions of x and y . Are the conditions sufficient? $5+1=6$

- (ii) Find the first *three* terms of the Taylor series expansion of the complex variable function

$$f(z) = \frac{1}{z^2 + 4} \quad \text{about } z = -i. \quad 4$$

- (b) Evaluate the following integrals using calculus of residues : **(any two)**

$$5+5=10$$

(i)
$$\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^2} dx$$

(ii)
$$\int_0^{2\pi} \frac{d\theta}{5 - 4 \sin \theta}$$

(iii)
$$\int_0^{\infty} \frac{\sin x}{x} dx$$

- (c) State and prove Fourier integral theorem.

- (d) (i) Applying change of scale theorem, find

$$L[\sin 3t]. \quad 2$$

- (ii) By the Laplace transform method, develop the formal solution of the differential equation which characterizes the motion of a damped harmonic oscillator. 8

- (e) (i) Show that $\frac{\partial x^p}{\partial x^q} = \delta_q^p$ 1

- (ii) Show that the components of Kronecker delta δ_j^i do not change under coordinate transformation. 4

- (iii) A covariant tensor has components $xy, 2y - z^2, xz$ in rectangular coordinates. Find its covariant components in spherical coordinates. 5

- (f) (i) Find the inverse Laplace transform

$$\frac{2s^2 - 4}{(s+1)(s-2)(s-3)} \quad 6$$

- (ii) State and prove the first shifting property of Laplace transform. 4