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3 (Sem-6/CBCS) MAT HC2

2025

MATHEMATICS

(Honours Core)

Paper : MAT-HC-6026

(Partial Differential Equations)

Full Marks : 60

Time : Three hours

***The figures in the margin indicate
full marks for the questions.***

1. Answer the following as directed : $1 \times 7 = 7$
 - (i) Which of the following methods can be used to construct a first-order partial differential equation ?
 - (a) By differentiating a given function with respect to multiple independent variables
 - (b) By eliminating one or more arbitrary constants from a given relation



- (c) By integrating a given function with respect to the dependent variable
- (d) None of the above
(Choose the correct answer)
- (ii) Along every characteristic strip of the equation $F(x, y, z, p, q) = 0$, the function $F(x, y, z, p, q)$ is _____.
(Fill in the blank)
- (iii) Charpit's method can be applied to both linear and nonlinear first-order partial differential equations.
(State True or False)
- (iv) What is the primary goal of transforming a first-order linear PDE into its canonical form?
- (a) To simplify the equation and make it easier to solve, often using characteristic curves
- (b) To eliminate the need for the method of characteristics
- (c) To ensure the equation has only one variable

- (d) To convert the equation into a second-order PDE.

(Choose the correct answer)

- (v) In the method of separation of variables, we assume a solution of the form $u(x, y) = X(x)Y(y)$, leading to two ODEs. The constant λ that arises from separation is known as the _____ constant.
(Fill in the blank)
- (vi) Which of the following is a characteristic of a hyperbolic second-order linear partial differential equation?
- (a) It describes steady-state phenomena
- (b) It describes systems in equilibrium
- (c) It models wave propagation
- (d) It has a solution that does not change over time

(Choose the correct answer)

- (vii) The general solution of a linear second-order partial differential equation with constant coefficients is the sum of the _____ (the solution to the corresponding homogeneous equation) and the particular integral (a solution to the non-homogeneous equation).

(Fill in the blank)

2. Answer in short : 2×4=8

- (i) Define first-order quasi-linear and semi-linear partial differential equations.
- (ii) Construct the first-order partial differential equation for the family of surfaces defined by $z = x^2 + y^2 + xy + C$, where C is a constant.
- (iii) State the basic idea behind Cauchy's method of characteristics for solving nonlinear first-order partial differential equations.
- (iv) Determine whether the following equation is parabolic, elliptic or hyperbolic.

$$u_{xx} + x^2 u_{yy} = 0$$

3. Answer **any three** : 5×3=15

- (i) Find the integral surface of the equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 1$.
- (ii) Define the concept of 'general integral' of a first-order nonlinear partial differential equation. Explain it for the equation $z^2(1 + p^2 + q^2) = 1$.
- (iii) Reduce to canonical form and find the general solution of $u_x + xu_y = y$.
- (iv) Apply $\sqrt{u} = v$ and $v(x, y) = f(x) + g(y)$ to solve the equation $x^4 u_x^2 + y^2 u_y^2 = 4u$.
- (v) Find the characteristic curves and then reduce the equation $u_{xx} + (2 \operatorname{cosec} y)u_{xy} + (\operatorname{cosec}^2 y)u_{yy} = 0$ to the canonical form.

4. Answer the following : 10×3=30

- (i) Find a complete integral of the equation $(p^2 + q^2)x = pz$ and deduce the solution which passes through the curve $x = 0, z^2 = 4y$.

Or

Solve -

$$(p_1 + x_1)^2 + (p_2 + x_2)^2 + (p_3 + x_3)^2 = 3(x_1 + x_2 + x_3)$$

by Jacobi's method.

- (ii) Apply the method of separation of variables $u(x, y) = f(x)g(y)$ to solve the equation $y^2u_x^2 + x^2u_y^2 = (xyu)^2$,

$$u(x, 0) = 3 \exp\left(\frac{x^2}{4}\right).$$

Or

Apply $v = \ln u$ and then

$$v(x, y) = f(x) + g(y) \text{ to solve the equation } x^2u_x^2 + y^2u_y^2 = (xyu)^2.$$

- (iii) Determine the region in which the given equation is hyperbolic, parabolic, or elliptic, and transform the equation in the respective region to canonical form.

$$(a) \quad u_{xx} + xyu_{yy} = 0$$

$$(b) \quad u_{xx} + u_{xy} - xu_{yy} = 0$$

Or

Find the general solutions of the following equations :

$$(a) \quad x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 0$$

$$(b) \quad 3u_{xx} + 10u_{xy} + 3u_{yy} = 0$$
