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3 (Sem–6/CBCS) MAT HE 5

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Nowgong Girls' College

2025

MATHEMATICS

(Honours Elective)

Paper : MAT–HE–6056

(Rigid Dynamics)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×10=10

   (a) Let $ABCD$ be the lamina such that $AB = 2a$ and $AD = 2b$. What is the moment of inertia of the lamina about an axis through its centre and perpendicular to its plane ?

   (b) Write down the moment of inertia of a hollow sphere of radius a about a diameter.

   (c) Write down the moment of inertia of a right circular cylinder about its axis.

(d) If $(x_i, y_i), (i = 1,2,...n)$, be the coordinates of the particles of masses $m_i \, (i = 1,2,....,n)$ referred to two mutually perpendicular lines *ox* and *oy*, then write down the product of inertia of the system of particles with respect to the lines *ox* and *oy*.

(e) Define momental ellipsoid of a rigid body at a point *O*.

(f) State D'Alembert's principle.

(g) If a rigid body swings , under gravity, then write down the time of a complete small oscillation.

(h) Define centre of oscillation.

(i) State the principle of conservation of angular momentum under finite forces.

(j) What is the degree of freedom of a rigid body which can move freely in space ?

2. Answer the following questions : 2×5=10

(a) What are the necessary and sufficient conditions for the two systems of bodies to be equimomental ?

(b) The lengths *AB* and *CD* of the sides of a rectangle *ABCD* are $2a$ and $2b$. Show that the inclination to *AB* of one of the principal axes at *A* is

$$\frac{1}{2}\tan^{-1}\left\{\frac{3ab}{2\left(a^2 - b^2\right)}\right\}.$$

(c) Consider a rigid body consisting of five particles m_1, m_2, m_3, m_4, m_5 having masses 1 units, 2 units, 3 units, 4 units, 5 units and located at the points (–1,0,1), (0,0,1), (1,0,1), (1,1,0), (–1,1,0) respectively. Find the products of inertia about

(i) *x-axis* and *z-axis*

(ii) *y-axis* and *z-axis*

(d) Find the length of the simple equivalent pendulum for the minimum time of oscillation of a compound pendulum.

(e) Find the kinetic and potential energy for a simple pendulum.

3. Answer ***any four*** questions : 5×4=20

(a) If the moments and products of inertia of a body about three perpendicular and concurrent axes are known, then find the moment of inertia about any other axis through their meeting point.

(b) Show that the momental ellipsoid at a point on the rim of a hemisphere is

$$2x^2 + 7\left(y^2 + z^2\right) - \frac{15}{4}xz = \text{constant}.$$

(c) A uniform rod AB is freely movable on a rough inclined plane whose inclination to the horizon is i and whose coefficient of friction is μ, about a smooth pin fixed through the end; the bar is held in the horizontal position in the plane and allowed to fall from this position, if θ be the angle through which it falls from rest show that $(\sin\theta / \theta) = \mu \cot i$.

(d) If the forces acting on a system be such that they have no moment about a certain fixed line throughout the motion then show that the moment of momentum about this straight line is constant.

(e) Use Lagrange's equations to find the differential equation for a compound pendulum which oscillates in a vertical plane about a fixed horizontal axis.

(f) A body, whose mass is m, is acted upon at a given point P by a blow of impulse X. If v and v' be the velocities of P in the direction of X just before and just after the action of X, show that the change in the kinetic energy of the body i.e, the work done on it by the impulse, is $\frac{1}{2}(v + v')X$.

4. Answer ***any four*** parts : 10×4=40

(a) (i) Find the moment of inertia of any triangular area ABC about a perpendicular to its plane through A. 5

(ii) Find the principal axes of a right circular cone at a point on the circumference of the base; and show that one of them will pass through its centre of gravity if the vertical angle of the cone is $2\tan^{-1}\left(\frac{1}{2}\right)$. 5

(b) Three rods AB, BC and CD, each of mass m and length $2a$, are such that each is perpendicular to the other two. Show that the principal moments of inertia at the centre of the mass are ma^2, $\frac{11}{3}ma^2$ and $4ma^2$. 10

(c) (i) Prove that the centre of inertia of a body moves as if the whole mass of the body were collected at it, and as if all the external forces were acting at it in direction parallel to those in which they act. 5

(ii) A rod of length $2a$, is suspended by a string of length l, attached to one end, if the string and rod revolve about the vertical with uniform angular velocity, and their inclination to the vertical be θ and ϕ respectively, show that

$$\frac{3l}{a} = \frac{(4\tan\theta - 3\tan\phi)\sin\phi}{(\tan\phi - \tan\theta)\sin\theta}.$$ 5

(d) A thin circular disc of mass M and radius a, can turn freely about a thin axis OA, which is perpendicular to its plane and passes through a point O of its circumference. The axis OA is compelled to move in a horizontal plane with angular velocity ω about its end A. Show that the inclination θ to the vertical of the radius of the disc through O is $\cos^{-1}\left(g/a\omega^2\right)$ unless $\omega^2 < g/a$ and then θ is zero. 10

(e) (i) Show that the centres of suspension and oscillation of a compound pendulum are convertible. 5

(ii) A solid homogeneous cone of height h and vertical angle 2α oscillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{1}{5}h\left(4+\tan^2\alpha\right)$. 5

(f) (i) Find how an equilateral lamina must be struck so that it may commence to rotate about a side. 5

(ii) A cylinder rolls down a smooth plane whose inclination to the horizontal is α, unwrapping, as it goes, a fine string fixed to the highest point of the plane; find its acceleration and the tension of the string. 5

(g) (i) Define generalised coordinates of a system. 2

(ii) Derive the Lagrange's equations of motion in generalised coordinates. 8

(*h*) A uniform rod is placed with one end in contact with a horizontal table, and is then at an inclination α to the horizon and is allowed to fall, when it becomes horizontal, show that its angular velocity is $\left(\frac{3g}{2a} \sin \alpha\right)^{\frac{1}{2}}$ whether the plane is perfectly smooth or perfectly rough. 10
