Total number of printed pages–7

1 (Sem–4) PHY 2

2025

PHYSICS

Paper : PHY0400204

(*Quantum Mechanics*)

Full Marks : 45

Time : 2 hours

The figures in the margin indicate full marks for the questions.

Symbols have their usual meanings.

1. Objective-type : (*Answer **all** questions*) 1×5=5

(*a*) If λ_c is the Compton shift, what is the greatest wavelength change in Compton scattering?

(*b*) What is the outcome of $\left[\hat{x}, e^{\hat{x}}\right]$?

(c) Plot the wavefunction $\psi(x) = \frac{1}{a^2} x e^{-x/a}$ for $x > 0$, where a is a constant and real number.

(d) What is the requirement of de Broglie wavelength of electron for the diffraction of electrons by a crystal?

(e) What is the total degeneracy in energy of H-atom with principal quantum number $n = 2$?

2. Very short answer-type : *(Answer **any five** questions)* 2×5=10

(a) Write Planck's blackbody radiation formula and obtain Rayleigh-Jeans formula under limiting condition.

(b) Write the general form of the eigenvalue equations for the Hamiltonian of a one-dimensional linear harmonic oscillator and mention possible eigenvalues.

(c) What is the minimum value of the product $\Delta x \Delta p_x$? Plot Δp_x versus Δx.

1+1=2

(d) Why group velocity and not the phase velocity is considered to describe the velocity of a moving material particle?

(e) What is zero-point energy? Why it cannot be equal to zero for a particle confined in a potential box?

1+1=2

(f) If λ_p and λ_α are the de Broglie wavelengths of a proton and an alpha particle moving with same non-relativistic speeds, then find the ratio $\frac{\lambda_p}{\lambda_\alpha}$.

(g) Write the differential forms of linear momentum and energy operators. Is momentum operator Hermitian?

1+1=2

(h) An attempt is made to measure the position of an electron in an atom. The uncertainty of this measurement is 1 Å. What is the minimum uncertainty in the measurement of linear momentum of the electron?

(i) Using the general expression for spherical harmonics $Y_\ell^{m_l}(\theta, \varphi)$, evaluate Y_1^0.

(j) Write the eigenvalue equations for the z-component of orbital angular momentum and square of the orbital angular momentum operators for a particle under the action of a spherically symmetric potential.

3. Short answer-type : *(Answer **any four** questions)*

5×4=20

(a) Obtain the normalization constant by normalizing the given wavefunction $\psi(x) = \frac{3}{\sqrt{10}}(a^2 - x^2)$ in the region $-a \le x \le a$. Hence, show the variation of the normalized wavefunction with x graphically with a mention of the peak value.

3+2=5

(b) An experiment on photoelectric effect is conducted for a metal. The stopping potentials are $4.50V$ and $0.20V$ corresponding to light wavelengths $190nm$ and $550nm$, respectively. Find the work function of the metal.

(c) Provide a brief physical interpretation of wavefunction.

(d) Consider a beam of particles of mass m, moving in the positive x direction with energy E towards a potential step at $x = 0$. The potential $V(x)$ is zero for $x \le 0$ and it is $\frac{3}{4}E$ for $x > 0$. Find the reflection coefficient.

(e) Starting with time dependent Schrödinger equation, obtain the differential form of equation of continuity involving the probability current density.

(f) Briefly describe the Davisson-Germer experiment that confirms wave nature of electrons.

(g) Write the time-independent Schrödinger equation for a one-dimensional linear harmonic oscillator and provide its ground state solution using the Hermite polynomials.

(h) Find the outcome of the commutation relation,

$$\left[\hat{x}\hat{p}_y - \hat{y}\hat{p}_x, \hat{y}\hat{p}_z - \hat{z}\hat{p}_y\right]$$

4. Essay-type : *(Answer **any one** question)*

10×1=10

(a) Starting with the concept of wave packet, obtain the intensity distribution. Introduce the Gaussian form of wave packet and briefly explain its connection with a moving material particle. 7+3=10

(b) Write the time independent Schrödinger equation in three dimensions for a particle experiencing central potential and obtain its radial and angular parts in spherical polar coordinate system. Using the idea of separation of variables, find the normalized azimuthal wavefunction. 7+3=10

(c) Consider a particle inside a one-dimensional potential box having infinite potential barriers at $x = 0$ and $x = L$. The wavefunction of the particle is $\psi(x) = Nx(L-x)$, where N is the normalization constant. Find the expectation values of position and linear momentum operators.
