2018

MATHEMATICS

(Major)

Paper : 2.1

(Coordinate Geometry)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) What is the locus represented by the equation

$$ax^2 + 2hxy + by^2 = 0?$$

(b) Write down the formulae of transformation from one pair of rectangular axes to another with the same origin in two dimensions.

(c) What is the eccentricity of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1?$$

(d) Write down the asymptotes of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(e) What is the equation of the tangent to the parabola $y^2 = 4ax$ at the point (x_1, y_1)?

(f) Write down the equation of z-axis in symmetrical form.

(g) What are the direction cosines of the normal to the plane given by the equation $ax + by + cz + d = 0$?

(h) Define skew lines.

(i) Write down the centre and radius of the sphere given by the equation

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

(j) Mention the condition under which the lines

$$\frac{x-\alpha_1}{l_1} = \frac{y-\beta_1}{m_1} = \frac{z-\gamma_1}{n_1}$$

and $$\frac{x-\alpha_2}{l_2} = \frac{y-\beta_2}{m_2} = \frac{z-\gamma_2}{n_2}$$

are coplanar.

2. Answer the following questions : 2×5=10

(a) Find the transformed equation of the line $y = x$ when the axes are rotated through an angle 45°.

(b) Find the angle between the pair of lines

$$ax^2 + 2hxy + by^2 = 0$$

the axes being rectangular.

(c) Mention the conditions under which the general equation of second degree

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents *(i)* a parabola, *(ii)* an ellipse, *(iii)* a hyperbola and *(iv)* a circle.

(d) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point (1, 2, 3).

(e) Obtain the equation of a cone with its vertex at the origin and passing through the curve $ax^2 + by^2 + cz^2 = 1$, $lx + my + nz = p$.

3. Answer any *two* parts : 5×2=10

(a) If by a rotation of the rectangular axes about the origin, the expression $ax^2 + 2hxy + by^2$ changes to

$$a_1x_1^2 + 2h_1x_1y_1 + b_1y_1^2$$

then prove that $a + b = a_1 + b_1$ and

$$ab - h^2 = a_1b_1 - h_1^2$$

(b) Show that the straight lines joining the origin to the other two points of intersection of the curves, whose equations are

$$ax^2 + 2hxy + by^2 + 2gx = 0$$

and $a_1x^2 + 2h_1xy + b_1y^2 + 2g_1x = 0$ will be at right angles if $g(a_1 + b_1) = g_1(a + b)$.

(c) Find the equation of the plane through the point (2, 3, 5) and parallel to the plane $2x - 4y + 3z = 9$.

(d) If P and Q are the extremities of a pair of conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then prove that the locus of the middle point of PQ is the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2}$$

4. Answer any *two* parts : 5×2=10

(a) Show that the equation of the tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$ at the point whose vertical angle is α is given by

$$\frac{l}{r} = e\cos\theta + \cos(\theta - \alpha)$$

(b) If the line $\frac{lx}{a} + \frac{my}{b} = n$ cuts the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at the ends of a pair of conjugate diameters, then prove that

$$l^2 + m^2 = 2n^2$$

(c) Find the lengths of the semi-axes of the conic $ax^2 + 2hxy + by^2 = 1$.

(d) Find the asymptotes of the hyperbola

$$2x^2 - 3xy - 2y^2 + 3x + y + 8 = 0$$

and derive the equations of the principal axes.

5. Answer any *four* parts : 5×4=20

(a) Find the equation of the polar of the point (2, 3) with respect to the conic

$$x^2 + 3xy + 4y^2 - 5x + 3 = 0$$

(b) A variable plane is at a constant distance p from the origin and meets the axes in A, B and C. Through A, B and C, planes are drawn parallel to the coordinate planes. Show that the locus of their point of intersection is

$$x^{-2} + y^{-2} + z^{-2} = p^{-2}$$

(c) Show that the shortest distance between any two opposite edges of the tetrahedron formed by the planes $y+z=0$, $z+x=0$, $x+y+z=a$, $x+y=0$ is $\frac{2a}{\sqrt{6}}$ and that the three lines of shortest distance intersect at the point $(-a, -a, -a)$.

(d) Show that by proper choice of axes, the equations of two non-intersecting straight lines can be put in the form $y = mx$, $z = c$ and $y = -mx$, $z = -c$.

(e) A variable plane is parallel to the given plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$$

and meets the axes in A, B and C. Prove that the circle ABC lies on the cone

$$yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$$

(f) Find the condition that the cone

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$$

may have three perpendicular generators.

6. Answer any *four* parts : 5×4=20

(a) Find the equation of the cylinder whose generators are parallel to the line $2x = y = 3z$ and which passes through the circle $y = 0$, $z^2 + x^2 = 8$.

(b) Prove that from any point, five normals can be drawn to the paraboloid

$$ax^2 + by^2 = 2cz$$

(c) Find the equation of the director sphere of the conicoid $ax^2 + by^2 + cz^2 = 1$.

(d) Tangent planes are drawn to the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

through (α, β, γ). Prove that the perpendiculars to them from the origin generate the cone

$$(\alpha x+\beta y+\gamma z)^2 = a^2x^2+b^2y^2+c^2z^2$$

(e) Prove that the plane $ax+by+cz=0$ cuts the cone $yz+zx+xy=0$ in perpendicular lines if

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$$

(f) Find the locus of chords of the conicoid

$$ax^2+by^2+cz^2=1$$

bisected at a given point (α, β, γ).

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