2016

MATHEMATICS

(Major)

Paper : 2·1

(Coordinate Geometry)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) What is the locus represented by the equation $x^2 - 5xy + 6y^2 = 0$?

(b) What will be the equation of the line $x + y = 2$, when the origin is transferred to the point $(1, 1)$?

(c) About which axis the parabola $y^2 = 4ax$ is symmetric?

(d) The parametric equations $x = a\sec\phi$ and $y = b\tan\phi$ represent *(i)* an ellipse, *(ii)* a parabola, *(iii)* a hyperbola.

Find the correct answer.

(e) What are the direction ratios of the normal to the plane $ax + by + cz + d = 0$?

(f) Find the centre and radius of the sphere

$$2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z + 3 = 0$$

(g) What is the general equation of a cone passing through the coordinate axes?

(h) Define skew lines.

(i) Write down the equation of the tangent to $\frac{l}{r} = 1 + e\cos\theta$ at α.

(j) The shortest distance between two lines is given to be zero. What conclusion can you make about the lines?

2. Answer the following : $2\times5=10$

(a) Transform the equation $x^2 - y^2 = a^2$ by taking the perpendicular lines $y - x = 0$ and $y + x = 0$ as coordinate axes.

(b) Find the equation of the plane through the point (2, 3, 5) and parallel to the plane $2x - 4y + 3z = 9$.

(c) If $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are the extremities of any focal chord of the parabola $y^2 = 4ax$, then prove that

$$t_1 t_2 = -1$$

(d) Find the equation of the plane containing the lines

$$2x+3y+5z-7=0$$
$$3x-4y+z+14=0$$

and passing through the origin.

(e) Find the equation of the right circular cone whose vertex is the origin, axis is the z-axis and semi vertical angle is α.

3. Answer any *two* parts : 5×2=10

(a) If by a rotation of the rectangular axes about the origin, the expression $ax^2+2hxy+by^2$ changes to $a'x'^2+2h'x'y'+b'y'^2$, then show that

$$a+b=a'+b'$$
$$ab-h^2=a'b'-h'^2$$

(b) Prove that the straight lines represented by the equation

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

will be equidistant from the origin if

$$f^4-g^4=c(bf^2-ag^2)$$

(c) Find the condition under which the equation

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

represents a pair of parallel straight lines.

4. Answer any *two* parts : 5×2=10

(a) Find the equation of the pair of tangents from (x', y') to the parabola $y^2 = 4ax$.

(b) Prove that the middle points of the chords of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

parallel to the diameter $y = mx$ lie on the diameter $a^2 my = b^2 x$.

(c) Prove that the equation of the polar of the origin with respect to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

is $gx + fy + c = 0$.

5. Answer any *four* parts : 5×4=20

(a) Reduce the equation

$$7x^2 - 2xy + 7y^2 - 16x + 16y - 8 = 0$$

to the standard form.

(b) Prove that the length of the focal chord of the conic

$$\frac{l}{r} = 1 - e\cos\theta$$

which is inclined to the axis at an angle α is $\dfrac{2l}{1 - e^2 \cos^2 \alpha}$.

(c) Find the locus of a point such that the sum of the squares of its distances from the planes

$$x+y+z=0,\ x-z=0,\ x-2y+z=0$$

is 9.

(d) Find the shortest distance between the lines

$$ax+by+cz+d=0=a'x+b'y+c'z+d'$$

and the axis of z.

(e) A plane meets the coordinate axes in *A*, *B*, *C* such that the centroid of the triangle *ABC* is the point (p, q, r). Show that the equation of the plane is

$$\frac{x}{p}+\frac{y}{q}+\frac{z}{r}=3$$

(f) Prove that the equation of the plane through the lines

$$x+y-2z+4=0=3x-y+2z+1$$

and parallel to the line

$$\frac{x+2}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$$

is $20x-8y+16z+3=0$.

6. Answer any *four* parts : 5×4=20

(a) Find the centre and radius of the circle

$$x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$$
$$x - 2y + 2z = 3$$

(b) Show that the equation of the cylinder whose generators are parallel to the line

$$\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$$

and guiding curve is

$$x^2 + 2y^2 = 1, \; z = 3$$

is

$$3(x^2 + 2y^2 + z^2) + 8yz - 2zx + 6x - 24y - 18z + 24 = 0$$

(c) Find the condition that the plane $lx + my + nz = p$ may be a tangent plane to the conicoid $ax^2 + by^2 + cz^2 = 1$.

(d) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the coordinate axes in A, B, C. Prove that the equation of the cone generated by the lines drawn from O to meet the circle ABC is

$$yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$$

(e) Find the equation of the director sphere of the conicoid $ax^2+by^2+cz^2=1$.

(f) Show that any normal to the conicoid

$$\frac{x^2}{pa+q}+\frac{y^2}{pb+q}+\frac{z^2}{pc+q}=1$$

is perpendicular to its polar line with respect to the conicoid

$$\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}=1$$

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